The Use of the Frequency-Dependent Finite-Difference Time-Domain Method for Induced Current and SAR Calculations for a Heterogeneous Model of the Human Body

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Abstract—**This paper describes the use of the previously formulated Frequency-Dependent Finite-Difference Time-Domain ((FD)^{2}TD) method for analysis of an anatomically based heterogeneous man model exposed to ultra-wide-band electromagetic pulse sources. The human tissues' electrical permittivities, $\varepsilon_i^*(\omega)$, are described by Debye equations with two relaxation constants, and the equation $D(t) = \varepsilon^*(\omega)E(t)$ is converted to a finite-difference equation along with the Maxwell's equations used by the standard FDTD method. Using a single run with a broad-band pulse excitation, the (FD)^{2}TD method is used to calculate mass normalized rates of energy deposition (specific absorption rates or SAR's) and induced currents in the man model over a broad band of frequencies. Time-domain coupling of a representative ultrashort pulse of submanosecond rise time and nanosecond pulse duration to the human body is also examined.**

I. Introduction

THE finite-difference time-domain (FDTD) method is becoming increasingly popular for numerical calculations of electromagnetic scattering and absorption in human models, both from the point of view of safety as well as for medical applications such as hyperthermia [1]–[7]. For these applications, the FDTD method has been found to be extremely versatile and has been used to calculate mass-normalized rates of absorption of electromagnetic energy (specific absorption rates or SAR's in watts per kilogram (W/kg)) or induced currents for spatially uniform or nonuniform incident fields (far field or near field). Incident fields may be continuous-wave sinusoidally varying (CW) or transient, such as those for an electromagnetic pulse (EMP).

A weakness of the conventional FDTD algorithm is that the dispersion of the tissues' dielectric properties is ignored, and frequency-independent properties are assumed. While this is permissible for continuous-wave or narrow-band irradiation, the results may be highly erroneous for short pulses which have ultra-wide bandwidths.

Two general approaches to a Frequency Dependent Finite-Difference Time-Domain ((FD)^{2}TD) method have been developed. One approach is to convert the complex permittivity from the frequency domain to the time domain and convolve this with the time-domain electric fields to obtain time-domain fields for dispersive material. This discrete time-domain convolution may be updated recursively for some rational forms of complex permittivity, which removes the need to store the time history of the fields and makes the method feasible. This method has been applied to materials described by a first-order Debye relaxation equation [8]–[10], a second-order Lorentz equation with multiple poles [13], and to a gaseous plasma [12].

A second approach is to add a differential equation relating the electric flux density $\boldsymbol{D}$ to the electric field $\boldsymbol{E}$ and solve this new equation simultaneously with the standard FDTD equations. This method has been applied to 1D and 2D examples with materials described by a first-order Debye equation or second-order single-pole Lorentz equation [11], [14], and to 3D sphere and homogeneous two-thirds muscle equivalent man model with properties described by a second-order Debye equation [15]. A different derivation but similar application of this approach is given in [16].

In this paper, this second (FD)^{2}TD approach is extended to the heterogeneous anatomically based model of the human body where the tissue properties $\varepsilon_i^*(\omega)$ are described by tissue-specific second-order Debye equations. Using a single run with a broad-band pulse excitation, layer-averaged specific absorption rates (SAR's) and induced currents at various frequencies are obtained by taking the Fourier components of the induced $\boldsymbol{E}$ fields. Coupling of a representative extremely short (ultra-wide-band) pulse to the human body is also examined.

II. The Differential-Equation-Based (FD)^{2}TD Method

The time-dependent Maxwell's curl equations used for the FDTD method are:

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} = -\mu \frac{\partial \boldsymbol{H}}{\partial t} \tag{1}$$

$$\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} \tag{2}$$

where the flux density vector $\boldsymbol{D}$ is related to the electric field through the complex permittivity $\varepsilon^*(\omega)$ of the local tissue by

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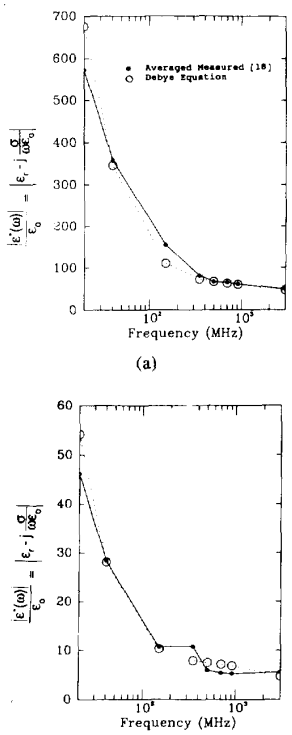

Fig. 1. Fit of Debye equation with two relaxation constants (4) to measured tissue properties of (a) muscle and (b) fat.

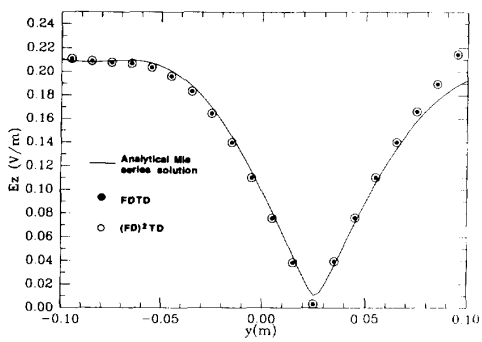

Fig. 2. Magnitude of E_z along the y-axis of a 2/3 muscle sphere at 200 MHz.

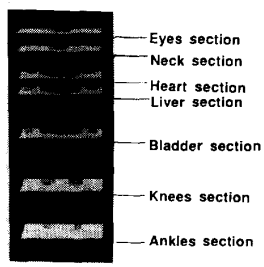

Fig. 3. Anatomically based heterogeneous man model with layers of interest highlighted.

the following equation:

$$\boldsymbol{D} = \varepsilon^*(\omega)\boldsymbol{E}. \tag{3}$$

Since (1) and (2) are solved iteratively in the time domain, (3) must also be expressed in the time domain. This may be done by choosing a rational function for $\varepsilon^*(\omega)$ such as the Debye equation with two relaxation constants

$$\varepsilon^*(\omega) = \varepsilon_0\left[\varepsilon_\infty + \frac{\varepsilon_{s1} - \varepsilon_\infty}{1 + j\omega\tau_1} + \frac{\varepsilon_{s2} - \varepsilon_\infty}{1 + j\omega\tau_2}\right]. \tag{4}$$

Rearranging (4) and substituting in (3) gives

$$\begin{aligned}\boldsymbol{D}(\omega) &= \varepsilon^*(\omega)\boldsymbol{E}(\omega) \\ &= \varepsilon_0\frac{\varepsilon_s + j\omega(\varepsilon_{s1}\tau_2 + \varepsilon_{s2}\tau_1) - \omega^2\tau_1\tau_2\varepsilon_\infty}{1 + j\omega(\tau_1 + \tau_2) - \omega^2\tau_1\tau_2}\boldsymbol{E}(\omega)\end{aligned} \tag{5}$$

where the dc (zero frequency) dielectric constant is given by

$$\varepsilon_s = \varepsilon_{s1} + \varepsilon_{s2} - \varepsilon_\infty. \tag{6}$$

Assuming $e^{j\omega t}$ time dependence, we can write (5) as a differential equation in the time domain

$$\begin{aligned}\tau_1\tau_2\frac{\partial^2\boldsymbol{D}}{\partial t^2} + (\tau_1 + \tau_2)\frac{\partial\boldsymbol{D}}{\partial t} + \boldsymbol{D} \\ = \varepsilon_0\left[\varepsilon_s\boldsymbol{E} + (\varepsilon_{s1}\tau_2 + \varepsilon_{s2}\tau_1)\frac{\partial\boldsymbol{E}}{\partial t} + \varepsilon_\infty\tau_1\tau_2\frac{\partial^2\boldsymbol{E}}{\partial t^2}\right].\end{aligned} \tag{7}$$

Equations (1), (2), and (7) can be converted from differential equations to finite-difference equations by approximating the space and time derivatives by the central difference formula. For details of this transformation, and other details of the basic FDTD method, the reader is referred to [17].

For the (FD)^{2}TD method, we solve (7) to find $\boldsymbol{E}$, (1) to find $\boldsymbol{H}$, and (2) to find $\boldsymbol{D}$ at each cell location. The $\boldsymbol{E} \rightarrow \boldsymbol{H} \rightarrow \boldsymbol{D}$ loop is then repeated until steady state is reached. The detailed procedure and the difference equations for (1), (2), and (7) are given in [15].

III. Modeling of Biological Tissue Properties with the Debye Equation

The measured properties of biological tissues (muscle, fat, bone, blood, intestine, cartilage, lung, kidney, pancreas, spleen, lung, heart, brain/nerve, skin, and eye) were obtained from [18]. Optimum values for ε_{s1}, ε_{s2}, ε_∞, τ_1, and τ_2 in (4) were obtained by nonlinear least squares matching to the measured data for fat and muscle. All other tissues have properties falling roughly between these two. Optimum values shown in Table I for ε_{s1}, ε_{s2}, and ε_∞ for all tissues were then obtained with τ_1 and τ_2 being the average of optimized values for fat and muscle. This was done to facilitate volume averaging of the tissue properties in cells of the heterogeneous man model. Having τ_1 and τ_2 constant for all tissues allowed linear (volume) averaging of the ε values for each tissue in a given cell to calculate ε values for that cell. The measured tissue properties and those computed from the Debye equation with τ_1 and τ_2 being the average of fat and muscle are shown in

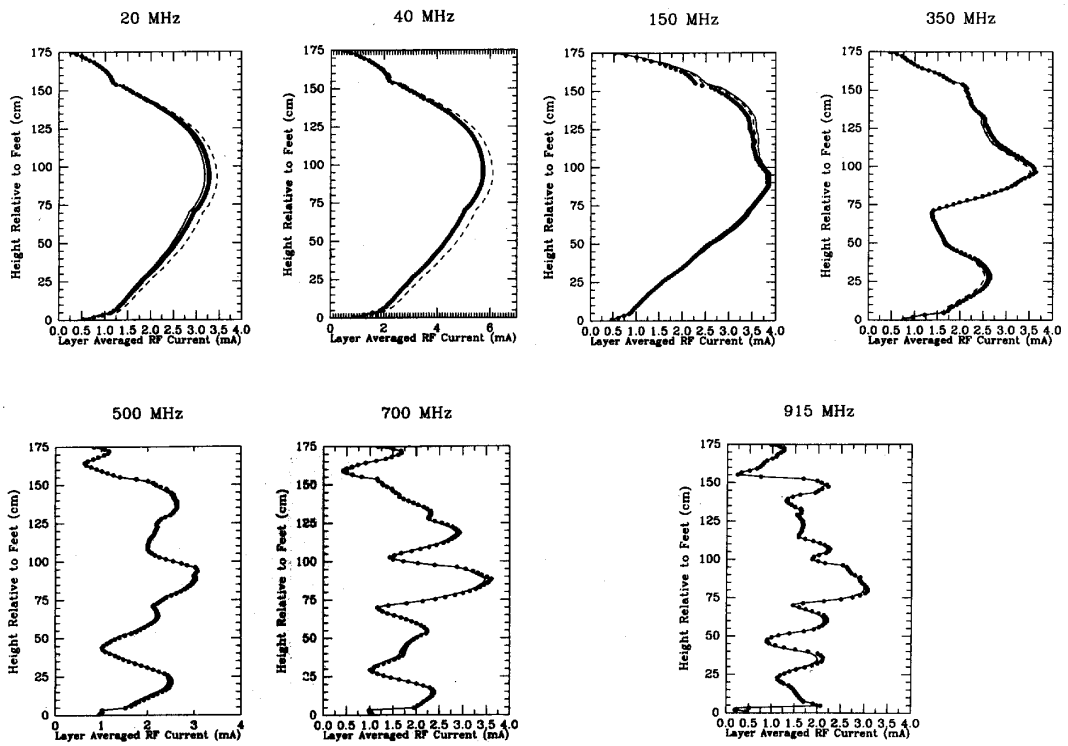

Fig. 4. Layer averaged RF current. Solid line: FDTD with exact properties; broken line: FDTD with 500-MHz properties. $(FD)^2TD$ (single run).

TABLE I
DEBYE CONSTANTS FOR TISSUES $\tau_1 = 46.25$ ns, $\tau_2 = 0.0907$ ns
(Average of optimum for fat and muscle.)

Tissue	ε_∞	ε_{s1}	ε_{s2}
Muscle	40.0	3948.	59.09
Bone / Cartilage	3.4	312.8	7.11
Blood	35.0	3563.	66.43
Intestine	39.0	4724.	66.09
Liver	36.3	2864.	57.12
Kidney	35.0	3332.	67.21
Pancreas / Spleen	10.0	3793.	73.91
1/3 Lung	10.0	1224.	13.06
Heart	38.5	4309.	54.58
Brain/Nerve	32.5	2064.	56.86
Skin	23.0	3399.	55.59
Eye	40.0	2191.	56.99

Fig. 1 for fat and muscle. Similar comparisons are observed for the other tissue types.

IV. VERIFICATION OF THE $(FD)^2TD$ METHOD

The $(FD)^2TD$ method is compared to analytical (Mie series) and FDTD methods for the test case of a 2/3 muscle sphere 20 cm in diameter for frequencies up to 350 MHz. The incident field is an E_x polarized plane wave traveling in the y direction. The time dependence of the incident field was a raised cosine pulse described by

$$p(t)\begin{cases} = [1 - \cos(2\pi f_{\max} t)], & 0 \le t \le 1/f_{\max} \\ = 0, & t \ge 1/f_{\max} \end{cases} \quad (8)$$

where $f_{\max} = 350$ MHz. The discrete Fourier transform was used to find the field distribution for several frequencies. These distributions were normalized to give results which would have been obtained using a 1-V/m incident plane wave with sinusoidal time variation at the frequency of interest. A cell size of 1 cm was used, giving a total $(FD)^2TD$ space of 38^3 cells. The magnitude of $E_z(t)$ along the y axis for 200 MHz is shown in Fig. 2. Other frequencies and components (E_x, E_y, etc.) also showed similar excellent agreement but are not shown for lack of space. The FDTD test was run using electrical properties of muscle calculated from the Debye equation at specific frequencies, so both FDTD and $(FD)^2TD$ used identical electrical properties and give virtually identical results, both in good agreement with the analytical (Mie series) solution.

V. APPLICATION OF THE $(FD)^2TD$ METHOD TO THE HETEROGENEOUS MAN MODEL

The $(FD)^2TD$ method is next applied to the heterogeneous man model previously used and described in [2]–[7]. The model is shown in Fig. 3 with several layers of interest highlighted. This anatomically based model is made up of 24 ×45× 135 1.31-cm (approximately 1/2-in) cells embedded in a total region of 42 ×63× 153 cells and requires 7.67 Mwords of memory and approximately 165 CPU minutes to run this test case on an IBM3090. By comparison, the traditional FDTD takes 6.37 Mwords and 155 CPU minutes, but must be rerun for each frequency. Memory and CPU time are minimized by using the limiting case of air-only FDTD equations in regions

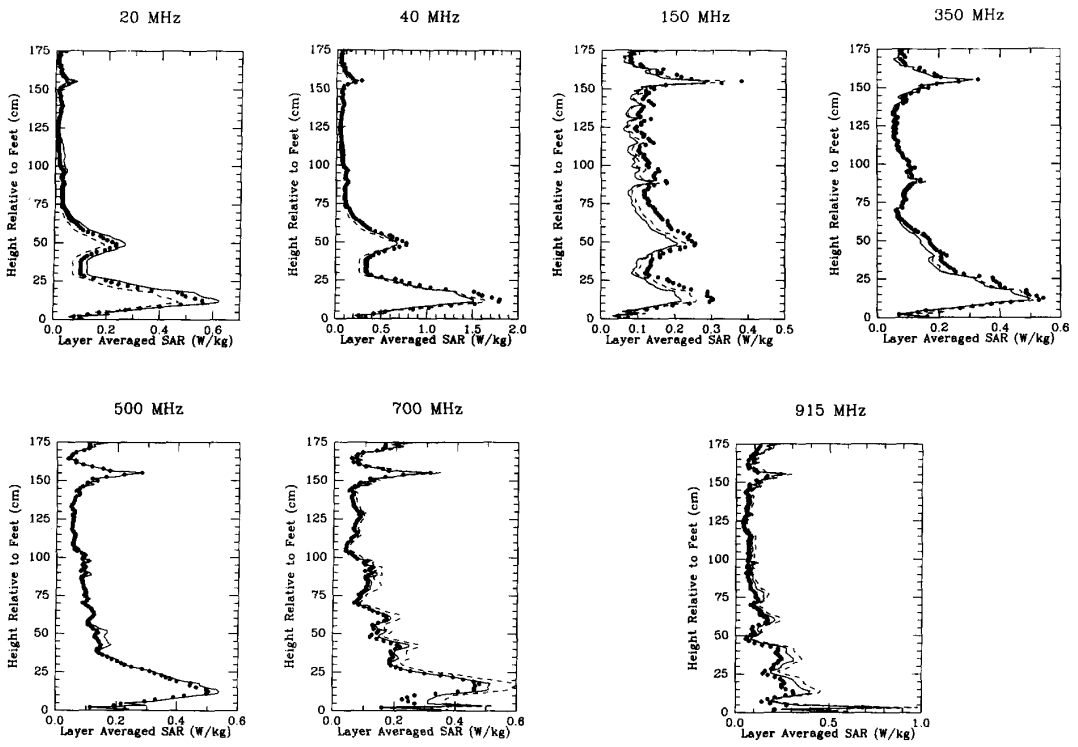

Fig. 5. Layer averaged SAR. Solid line: FDTD with exact properties; broken line: FDTD with 500-MHz properties. $(FD)^2TD$ (single run).

external to the body. Complex dielectric properties in each cell of the body are obtained by volume averaging properties for all tissues in each cell as described in Section III. The incident plane wave has the raised cosine pulse time duration given in (9) where $f_{\max} = 3$ GHz. It is E_z polarized (parallel to the long axis of the body), and is frontally incident on the body.

The layer averaged vertical current distribution is calculated from

$$I_z(t) = dx^2 \sum_i \sum_j \frac{\partial D_z(i,j)}{\partial t} \tag{9}$$

and the layer averaged specific absorption rate in the kth layer is calculated from

$$\mathrm{SAR}_k = \frac{1}{N_k} \sum_i \sum_j \frac{\sigma_{ijk}}{\rho_{\mathrm{ijk}}} \left[\frac{|E_x|^2 + |E_y|^2 + |E_z|^2}{2} \right]_{ijk} \tag{10}$$

where

dx cell size (m),
σ_{ijk} electrical conductivity (S/m) in the ijkth cell,
ρ_{ijk} mass density (kg/m^3) in the ijkth cell,
N_k number of tissue cells in the kth layer.

Layer-averaged current and SAR distributions computed using both FDTD and $(FD)^2TD$ methods are shown in Figs. 4 and 5, respectively, for frequencies from 20 to 915 MHz. Unlike the sphere test case, slight differences are observed between results computed using FDTD and $(FD)^2TD$. This is because the tissue properties for FDTD runs were chosen to be those given in [18] for each specific frequency, which differ slightly from those obtained from the Debye equation as shown in Fig. 1. The better the fit of the Debye equation (4) to the measured tissue properties, the better the aggreement will be between FDTD and $(FD)^2TD$. Notably, in spite of the slight mismatch between the Debye equation and measured tissue properties, the results for $(FD)^2TD$ are better than for FDTD with properties taken at the average frequency of 500 MHz, which would be a natural approximation to obtain results from a single run if $(FD)^2TD$ were not available.

VI. Coupling of an Ultra-Wide Band Pulse to the Human Body

Coupling of an ultra-wide band pulse to the heterogeneous model of the human body was also examined, since such pulses are being considered by the Department of Defense for future systems. The FDTD method is not able to accurately predict the time-domain behavior of ultra-wideband pulses in highly dispersive media, but the $(FD)^2TD$ method is well suited to this application. To demonstrate this, the body was illuminated by a vertically polarized raised cosine pulse described in (4) with $f_{\max} = 915$ MHz. The ground effect was neglected, and the body was assumed to be isolated from the ground. Fig. 6 shows the layer-averaged current passing through different layers of the body as a function of time. Fig. 7 shows the peak layer averaged currents for all layers of the body.

This test case illustrates the power of the $(FD)^2TD$ method. This new method provides accurate broad band and time domain data for a highly dispersive model from a single run,

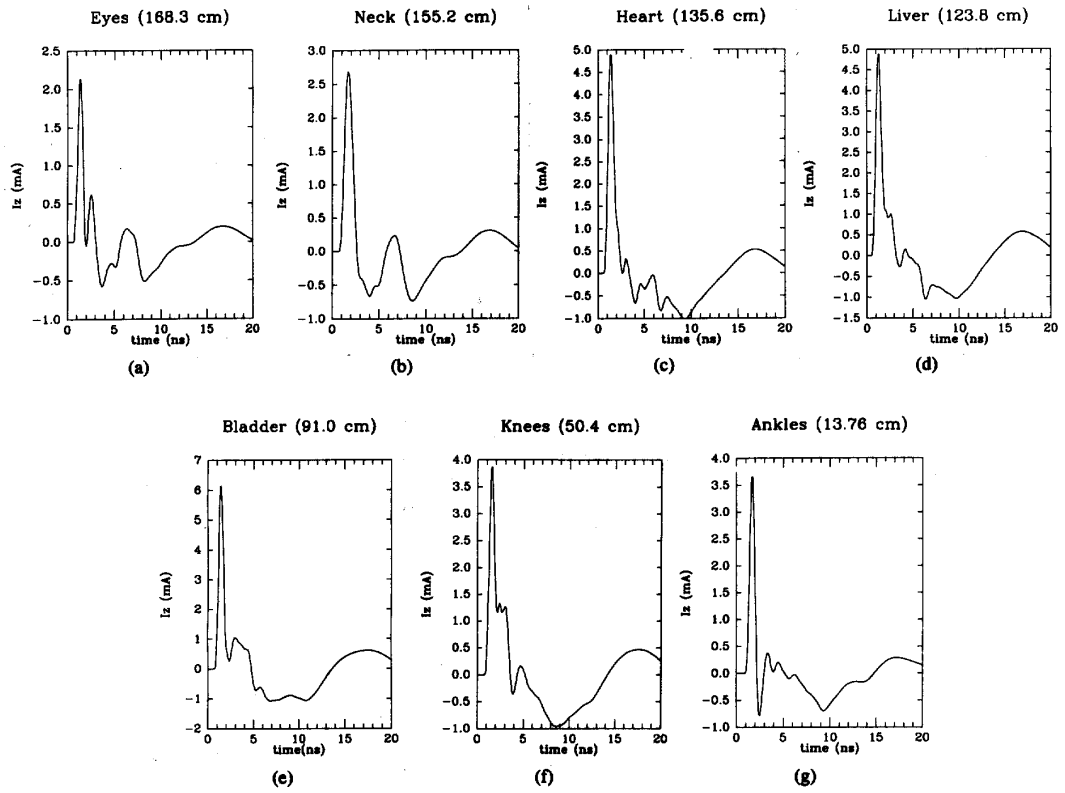

Fig. 6. Layer averaged RF current as a function of time. (a) Eye layer; height above feet = 168.3 cm. (b) Neck layer; height above feet = 155.2 cm . (c) Heart layer; height above feet = 135.6 cm. (d) Liver layer; height above feet = 123.8 cm. (e) Bladder layer; height above feet = 91.0 cm. (f) Knee layer; height above feet = 50.4 cm. (g) Ankle layer; height above feet = 13.76 cm.

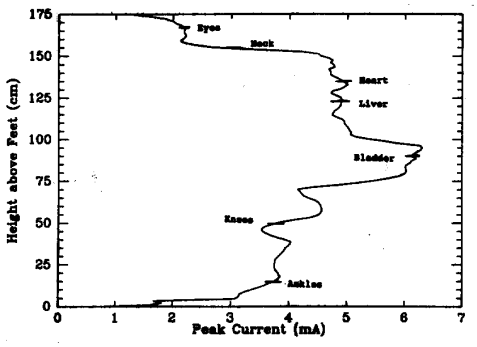

Fig. 7. Peak layer averaged RF current.

while the traditional FDTD method required a run at every frequency of interest and could not provide accurate time domain data for this dispersive model.

VII. Conclusion

This paper describes the application of the Frequency Dependent FDTD Method ((FD)^{2}TD) to a heterogeneous anatomically based model of the human body where the tissue properties $\epsilon_i^*(\omega)$ are described by Debye equations with two relaxation constants. Using a single run with a broad-band pulse excitation, the (FD)^{2}TD method provided SAR's and induced currents in either the time or frequency domain for this highly dispersive model.

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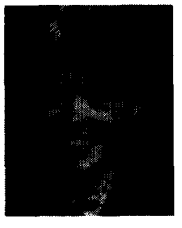

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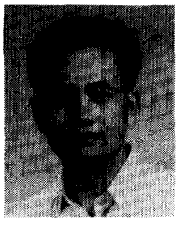

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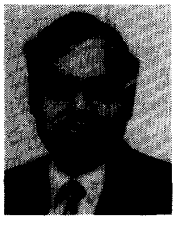
Dr. Chen is a member of the Editorial Board of IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES.



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